

Two-sided Search in International Markets

(preliminary draft)

J. Eaton^{a,b}, D. Jinkins^c, J. Tybout^{a,b}, D. Xu^{d,b1}

^aPenn State U., ^bNBER, ^cCopenhagen Business School, ^dDuke U.

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1 Introduction

To break into global markets, either as an exporter or an importer, firms must first identify foreign business partners. And since most international partnerships are short-lived, trading firms must continually seek new connections if they wish to maintain or expand their foreign market presence. The resulting patterns of international buyer-seller connections are surprisingly fluid, and they largely determine the dynamics of firm-level trade flows.

Herein we develop a new empirical model of these search and matching processes, quantify the associated costs, and explore their implications for trade dynamics and welfare. Specifically, we develop a dynamic model of trade in consumer goods with three types of agents: foreign exporters, domestic retailers, and domestic consumers. Heterogeneous exporters and retailers engage in costly search for one another, taking stock of their current situation and the structure of the buyer-seller network. The resulting matching patterns determine which retailers carry which varieties of goods. Consumers then choose how to allocate their expenditures across retailers and the individual goods that they offer. When a retailer and an exporter form a new business relationship, they divide their associated rents in a forward-looking Nash bargaining game, thereby determining the wholesale prices at which trade occurs. The retailer then passes the goods on to domestic consumers after adding an optimal mark-up.

Fit to customs records on Colombian footwear imports, our model speaks to a variety of empirical issues.¹ First, it provides estimates of the value of international business connections for different types of agents with different portfolios of business partners. Second, it allows us to decompose trade and welfare changes into two basic driving forces: market entry by

¹An application to U.S. apparel importers is in progress.

Chinese firms, and reductions in search costs. Similarly, it quantifies the capital gains and losses induced by these two types of shocks for different types of firms. Third, it characterizes the effects of search costs and foreign competition on firm dynamics. Finally, since firms with more clients find it less expensive to meet additional business partners, and since the rate at which firms acquire connections is partly due to luck, it quantifies the extent to which large firms owe their success to fortuitous events early in their life cycles.

Our model is related to a wide variety of earlier contributions. First, speaking broadly, it follows in the tradition of papers that analyze firm-to-firm matching in international markets, beginning with work by Rauch (2001), Rauch and Trindade (2002), and Rauch and Watson (2003). Some of these studies explores the implications of firms' uncertainty regarding the appeal of their products to foreign buyers (Rauch and Watson, 2003; Albornoz *et al.*, 2012; Eaton *et al.*, 2014), the prices that prevail in a foreign location (Allen, 2014; Steinwender, 2014; Bernard *et al.*, 2014a), or the characteristics and coordinates of potential foreign clients (Albornoz *et al.*, 2012; Rauch and Watson, 2003; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Antras and Costinot, 2011; Fernandez-Blanco, 2012). Other firm-to-firm trade models presume full information, and focus instead on the question of who matches with whom (Sugita *et al.*, 2014; Bernard *et al.*, 2014b; Lim, 2016).² Our model draws features from both strands of the literature. It incorporates uncertainty inasmuch as agents on each side of the market are unable to observe the characteristics and coordinates of potential business partners before they meet each other. And it generates assortative matching in a way similar to Bernard *et*

²There is also a large literature on value chains that treats firm-to-firm matching patterns (e.g., Antras and Shor (2013)). It is less related to our work in that it focuses on agency issues and sequences of upstream-downstream relationships.

al. (2014b): high-quality agents choose to search more intensively, and thus end up matching with a broader spectrum of partner types.

Second, our model resembles a number of recent trade papers in its emphasis on customer accumulation as a driver for firm dynamics (Albornoz *et al.*, 2012; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Chaney, 2014; Piveteau, 2015; Fitzgerald *et al.*, 2016).³ We depart from these papers by treating both exporters and importers as choosing their search intensity optimally. This formulation better conforms to actual practices, and allows us to generate richer exporter-importer network structures than would have been possible with a one-sided search model.

By making retailers a central feature of our model, we also contribute to the literature on intermediated trade. This includes papers that predict which kinds of exporters will use intermediaries (Blum *et al.*, 2009; Ahn *et al.*, 2011), and more relevant to our work, papers on the effects of trade on welfare under different types of intermediation and bargaining (Rauch and Watson, 2004; Antras and Costinot, 2011; Fernandez-Blanco, 2012; Bernard and Dhingra, 2015). Among these latter papers, the one most closely related to ours is Bernard and Dhingra (2015). Therein, exporters bargain with retailers abroad in order to avoid double marginalization and (in some cases) the price-depressing effects of competition among retailers. We too invoke a Nash bargaining game between retailers and exporters, but our focus is not on the endogenous choice of contract form.

Finally, we contribute to the literature on the life-cycle of exporters and importers. As

³Interest in this approach to firm dynamics is not confined to the trade literature. Recent contributions that focus on the accumulation of domestic customers include Foster *et al.* (2015) and Gourio and Rudanko (2014).

with the earlier literature on firm dynamics, our model is partly motivated by the "fat" tails that typically characterize firm-size distributions. One way earlier studies have generated these tails is through stochastic shocks to firm productivity or demand (Luttmer, 2007, 2011; Arkolakis, 2016). Another possibility for generating fat tails is to use a matching model and a convenient search cost function (Eaton *et al.*, 2014). We follow the latter modeling strategy. In particular, as in Eaton *et al.* (2014), we allow a firm's cost of finding new business partners to fall as the number of its current clients increases.

2 Data and Stylized Facts

Our modelling choices are partly motivated by the stylized facts that have recently emerged concerning international links between buyers and sellers. Studies reporting such facts are now available for the United States and Colombia (Eaton *et al.*, 2008, 2014; Bernard *et al.*, 2014b), Chile (Blum *et al.*, 2010), Mexico (Sugita *et al.*, 2014), Norway (Bernard *et al.*, 2014b), and Ireland (Fitzgerald *et al.*, 2016).

Below we present these facts for the population of Colombian firms that import footwear and their suppliers abroad. This choice of network reflects several considerations. First, this is a product category that foreign suppliers played an increasingly dominant role (50 - 60% market penetration) during our sample period, making trade networks important for domestic consumer welfare. Second, by choosing a sector in which most of the importers are wholesale/retail firms, we are able to keep the buyer side of the market relatively tractable. That is, within each wholesale or retail firm, revenue functions are nearly separable across categories of consumer goods, and firms' payoff functions can be reasonably approximated

with relatively simple expressions.

2.1 Data Sources

We base our analysis on data obtained from the Colombian customs authority: Dirección de Impuestos y Aduanas Nacionales de Colombia (DIAN). These data describe all merchandise shipments to Colombia. Each record includes a ten-digit Harmonized Schedule (HS) product code, shipment value, shipment quantity, entry or exit port, date of transaction, mode of transportation (land, sea, air), and the domestic firm’s tax identification number (NIT). Critically for our study, each record also includes the name and address of the foreign firm that is party to the transaction.

In order to keep track of foreign suppliers, we construct an alphanumeric foreign exporter ID for each shipment in the database. It is based on the business names and addresses that appear in the customs records. For example, one version of this ID combines the firm’s country code, first three letters of the first two main words in the firm’s name, the street address, and the first three letters of the city name.⁴ These codes are imperfect identifiers because, despite standardization, the same firm may appear in different records with slight differences in its spelling or address. The longer the string identifier, the more frequently this problem is likely to occur. On the other hand, when short string IDs are used, firms with names and street addresses that begin the same may become indistinguishable. Robustness checks and visual inspections of the data (in progress) will give us a sense for the importance of this issue.

⁴Before constructing strings, names and street addresses were standardized using the Stata routines "stnd_compname" and "stnd_address." Information on zip codes and states was also used in variants of the ID string.

2.2 Stylized Facts: Aggregates

We now document some stylized facts that motivate our model, focusing on the period 2006-2012.⁵ Table 1 reports time series on the total number of Colombian footwear importers, the total number of footwear exporters serving the Colombian market, the number of importer-exporter matches, and the total volume of imports in millions of dollars.

Note first that the aggregates are fairly stable during 2006 – 2009, so this period serves as a good benchmark. But thereafter Colombian imports grew rapidly, as did the number of matches and the number of exporters supplying the Colombian market. Further, total exports grew more rapidly than the number of exporters or the number of matches, so the typical exporter to Colombia increased its sales per business partner between 2010 and 2012.

The country’s rapid post-2009 import growth traces to three main factors. First, Colombia unilaterally reduced the import tariff for a broad range of manufacturing goods during 2010. For instance, the average import tariff for footwear was reduced from 13 percent in 2009 to 6 percent by the end of 2010. Second, Colombia had restrictions of ports of entry for Chinese and Panama products of textile, garments, and footwear during 2006 to 2009.⁶ The WTO ruled against this restriction at the end of 2009. In 2010, Colombia conformed to the WTO dispute settlement and removed its restrictions completely. Finally, Colombia also aggressively negotiated the formation of free trade areas (FTAs) with a few countries, most notably with

⁵We begin our sample in 2006 because there is a large drop of the number of importers from 2005 to 2006 with no large economic shocks. This is mostly due to the change of the registration system of importers for the textile/footwear (Decree 1299 of 2006).

⁶Colombia required the textiles, garments, and footwear products originating in or arriving from Panama or China to be imported exclusively through the Bogotá airport or the Barranquilla seaport.

Year	Importers	Exporters	Matches	Volume
2006	376	547	1232	158
2007	389	619	1386	199
2008	406	610	1412	239
2009	438	600	1404	241
2010	486	738	1673	304
2011	601	879	2097	452
2012	594	917	2104	540

Table 1: Number of Importers, Exporters, and Matches

Year	Countries of Origin		Countries of Exporting		
	CN/VNM/IDN	BRA/ECU	CN/HK	PAN	BRA/ECU
2006	71.3%	22.4%	30.3%	38.1%	22.8%
2007	71.0%	21.2%	35.8%	31.3%	21.2%
2008	74.2%	17.6%	33.5%	34.1%	17.6%
2009	75.5%	17.5%	29.4%	41.8%	17.6%
2010	79.2%	14.4%	31.9%	42.5%	14.4%
2011	81.7%	12.0%	29.7%	46.2%	12.0%
2012	84.9%	9.5%	34.2%	44.6%	9.5%

Table 2: Major Countries of Sellers

U.S. in 2011⁷.

We next break down Colombian footwear imports by exporting country. Table 2 reports time series on the share of aggregate import values accounted for by the main countries that supply Colombian footwear retailers and distributors.

On the right panel, we report the share of major “countries of exporting”, defined as the country where direct seller to Colombian importers is located. Here we note that, the largest surge in export values came from Panama. This is because Panama operates the largest free trade zone in the western hemisphere, and Asian manufacturers of consumer goods

⁷Starting 2013, under the pressure of domestic producers, Colombia imposed a \$5 per pair specific levy on all non-FTA countries and effectively reversed the trend.

have routinely used the trading companies located therein to reach the Colombian market.⁸ China/Hong Kong also account for substantial share based on “countries of exporting” and it remains stable over our sample period. On the other hand, Brazil and Eduardo rapidly loses market share to Panamanian firms, especially after 2009.

On the left panel, we report the share of major “countries of origin”, defined as the country where the products are manufactured. Here we observe a dominant role by low-cost Asian producers from China, Vietnam, and India. Combined they account for around 71 – 75% of total Colombia footwear import during 2006 - 2009. This number increased to 85% after 2009. Accordingly, while direct exports from Asia to Colombia did not grow particularly rapidly after 2009, low-cost footwear from China, Vietnam, and India also account for a substantial fraction of the post-2009 Panama import surge.

2.3 Stylized Facts: Buyer-Seller Distributions

We next exploit our buyer and seller IDs to summarize the frequency distributions of the sellers (a.k.a. exporters) per buyer (a.k.a. importer) and buyers per seller for the Colombian footwear’s international market.⁹ Figure 1 and Figure 2 depict the degree distributions for the three main HS4 categories of shoes: rubber, leather, and textile. On the horizontal axis,

⁸Colombian customs records show both the “country of exporting” and the “country of origin,” so it is possible to determine where imports arriving in Colombia from Panama “last underwent substantial transformation.” However, since Panamanian trading companies typically take ownership of the goods they sell to Colombian importers, the names of the Asian manufacturing firms do not appear on the invoices of the Asian-made goods that arrive in Colombia via Panama. This means we must treat the Panamanian firms as the exporters searching for business partners when we fit our model to the data.

⁹We will use “buyer” interchangeably with “importer” and “exporter” interchangeably with “seller.”

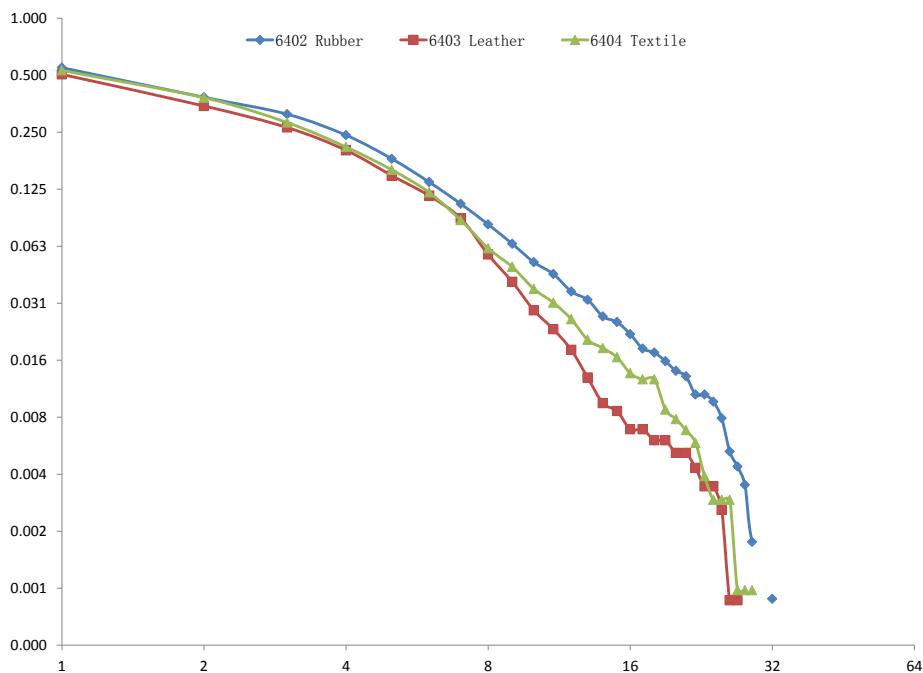


Figure 1: Degree distribution: sellers per buyer, 2006-2009

we have the number of connections of a buyer or seller. On the vertical axis, we report the inverse empirical CDF. Both axes are in log scale, so if the data were distributed according to a "power law" the lines would be linear.

Several patterns emerge. First, the distributions are quite similar for all types of footwear, so we will not be emphasizing cross-product distinctions much hereafter. Second, the tail of the distribution of sellers per buyer in Figure 1 begins to curve downward almost immediately, suggesting that no portion of the distribution is well-approximated by the Pareto distribution. Finally, the distribution of buyers per seller (Figure 2) is approximately power law in the left-hand tail. That is, out to about 20 buyers, the distribution is roughly Pareto.

Have these shapes changed over time? Figure 3 reports the inverse CDF of the number of sellers per buyer for both 2006 – 09 and 2010 – 12. The curve appears to be flattened out at

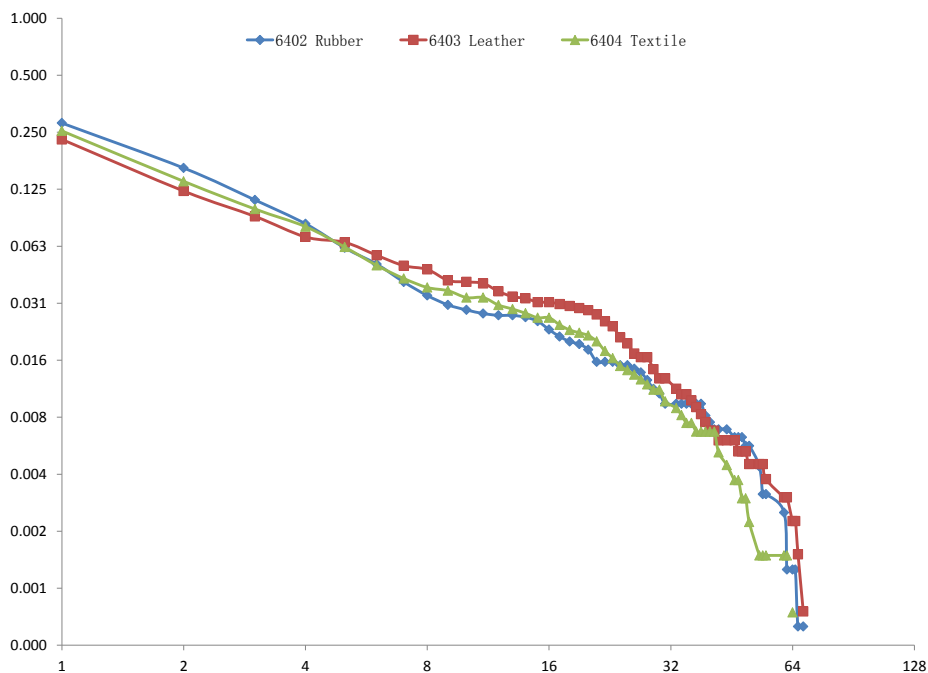


Figure 2: Degree distribution: buyers per seller, 2006-2009

right end of the degree distribution (>15) when trade has increased, indicating that there are relatively more large buyers.

Given the highly skewed distributions for both sellers per buyer and buyers per seller, it is natural to ask how important are the "power" buyers and sellers (i.e. those who transact with 5 or more business partners) in terms of aggregate imports. Table 3 shows the share of

# sellers	Frequency	Share of Imports
1	0.489	0.051
2	0.160	0.034
3	0.074	0.063
4	0.052	0.036
5	0.050	0.055
6 -10	0.118	0.384
10+	0.058	0.378

Table 3: Import Shares by Size of Buyer

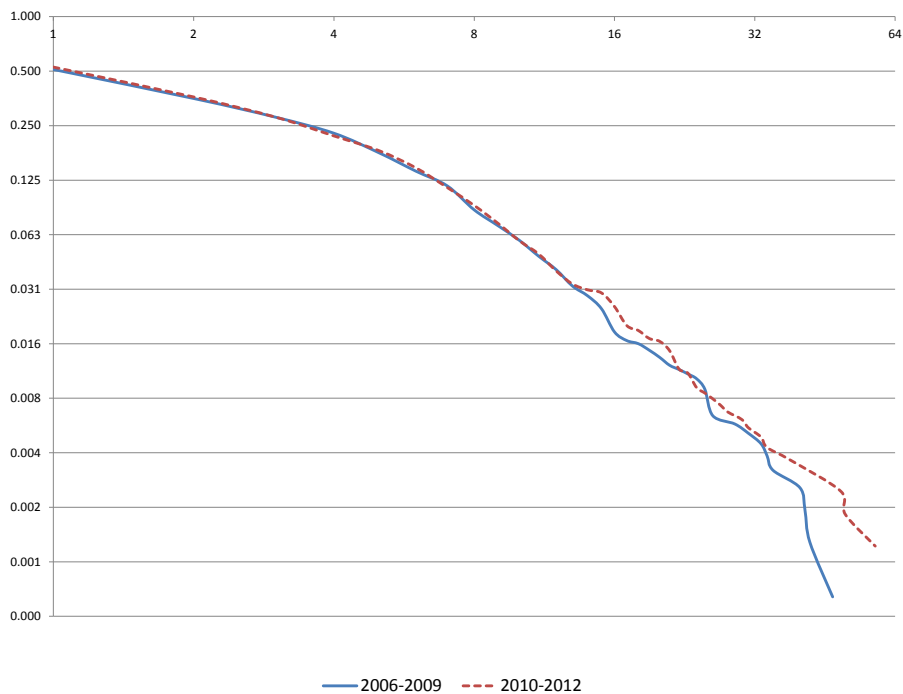


Figure 3: Degree distribution: sellers per buyer overtime

aggregate imports accounted for by Colombian importers with different numbers of business partners. Note that despite accounting for only 12 percent of the total number of importers, the power buyers account for 76 percent of aggregate imports. Thus, changes at the tail of the degree distribution are particularly important for industry aggregates and welfare.

2.4 Stylized Facts: Transition and Match Dynamics

Finally, given that our model will generate predictions on firm-level matching dynamics, it is useful to examine the overtime transitions rates for seller counts. We report these in Table 4. Several patterns are worth highlighting here. First, there is a non-trivial probability that one buyer's connections get completely eliminated from one year to the next. Some of these transitions to zero sellers reflect exit of the retailer, but most occur because the retailer

$t \setminus t+1$	0	1 to 5	6 to 10	10 to 15	15+
1 to 5	0.240	0.707	0.047	0.005	0.001
6 to 10	0.071	0.277	0.511	0.128	0.014
11 to 15	0.053	0.053	0.289	0.421	0.184
15+	0.097	0.000	0.065	0.194	0.645

Table 4: Transition Matrix of Sellers per Buyer

		Buyer		Seller	
Age	# Sellers	Cumulative # of Sellers	# Buyers	Cumulative # of Buyers	
2	0.59	1.59	0.21	1.22	
3	0.79	2.77	0.29	2.41	
4	1.23	4.89	0.45	4.48	

Table 5: Growth Relative to Birth

stopped stocking imported shoe varieties. Second, there is a general tendency for buyers to lose suppliers, on net. This is implied by the fact that, for any row, the probability mass to the left of the diagonal exceeds the mass to the right. Overall, this transition pattern is consistent with the cross-sectional distribution, in that both reflect a large probability mass at the lower numbers of connections.

Consistent with the previous table of transition, the life-cycle pattern of individual buyer and seller in terms of their business partner matches also involve active addition and destruction. Table 5 shows that for a new buyer starting to import, the average number of sellers it sources from goes up by 0.59 at age 2, 0.79 at age 3, and 1.23 at age 4. When we look at the unique number of sellers the buyer has cumulatively sourced from, though, the growth is much steeper and reaches 4.89 at age 4. This is consistent with the quick transition of the buyer-seller relationships. The average duration of a match in our data is 455 days (i.e. 1.25 years). The pattern is very similar from the seller’s perspective.

3 A model of buyer-seller networks

Motivated by the stylized facts described above, we now develop a continuous-time two-sided search model. As depicted in Figure 4, our model is populated by three types of agents: sellers, buyers, and consumers. Sellers provide goods to buyers in the international trade market, who pass them on to consumers in the retail market. Though we are thinking of sellers as foreign merchandise exporters and buyers as the domestic retailers they supply, our model could be applied in contexts that do not involve international trade.

Consumers acquire goods exclusively through retailers, who offer different but possibly overlapping menus of products, depending upon the set of suppliers they are currently partnered with. Retailers are also different in terms of the amenities they offer, like locational convenience, ambiance, and service. Consumers allocate their expenditures across retailers in a way that reflects their preferences for amenities and product menus.¹⁰

The dimensions of retailer heterogeneity are publicly observable, so consumers' expenditure patterns are characterized by a standard static optimization problem with full information. However, buyers and sellers in the wholesale market are unable to costlessly match with one another. Rather, each type of agent must invest in costly search to establish new business partnerships.

Because it is costly to find new business partners, buyers and sellers create rents when they meet one another. They bargain continuously and bilaterally over these rents, and the expected outcomes of these bargaining games determine the expected returns to successful

¹⁰Our setup draws equivalence to a nested Logit discrete choice model where indirect utility function of individual consumers depends on log price, see Verboven (1996)

search for each party.

Other things equal, the more intensively an agent searches, the higher the hazard rate with which she finds new partners and reaps her share of the associated rents. But these hazard rates depend upon other things as well.

First, matching hazards are influenced by market tightness. For example, when many buyers are searching for new suppliers, but not many suppliers are searching for new buyers, matching hazards will tend to be low for buyers and high for suppliers. As we will discuss shortly, the precise way in which search intensities on both sides of the market influence aggregate market tightness is determined by the matching function in our model, which we adopt from the labor-search literature.

Second, the ease with which agents find new business partners depends upon their previous successes. That is, agents who have already accumulated a large portfolio of business partners find it relatively easy to locate still more. This feature of our model, taken from Eaton *et al.* (2014), helps us to capture the "fat-tailed" distributions of buyers across sellers and sellers across buyers discussed above.

3.1 The Retail Market

Preferences and pricing: We now turn to model specifics. As in Akin *et al.* (forthcoming) and Bernard and Dhingra (2015), we start from a nested CES demand structure in which consumers have preferences over retailers, and within retailers, over products. Specifically, assume the retail market is populated by a measure- B continuum of stores, and suppose consumers view these stores as imperfect substitutes, both because they offer distinct amenities

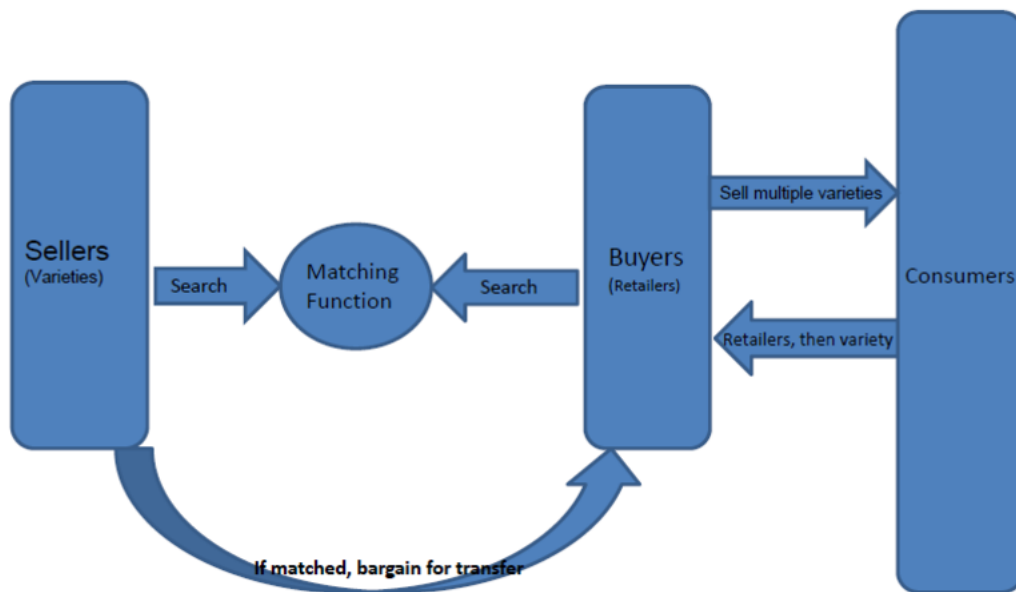


Figure 4: Model diagram

and because they carry different—but not necessarily disjoint—sets of products. More precisely, indexing stores by b and products (or exporting firms) by x , let consumers' preferences over retailers be given by the utility function:

$$C = \left[\int_{b \in B} (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where C_b measures consumption of the set of products, J_b , offered at store b ,

$$C_b = \left[\sum_{x \in J_b} (\xi_{xb} C_b^x)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}},$$

and μ_b and ξ_{xb} are exogenous parameters that measure the inherent appeal or quality of retailer b and product x , respectively.¹¹ This characterization of preferences implies that the

¹¹Alternative nesting structures are possible. In particular, consumers might have preferences over bundles of types of goods, each of which is a CES aggregation over the bundles available from alternative retailers.

exact price index for retailer b is $p_b = \left[\sum_{x \in J_b} \left(\frac{p_{xb}}{\xi_{xb}} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ and the exact price index for retailers as a group is $P = \left[\int_b \left(\frac{p_b}{\mu_b} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$.

Because of search frictions, retailers cannot instantaneously adjust the set of products they offer consumers. Rather, at each point in time they take their current offerings as given and engage in price competition in the retail market. It follows that the optimal retail prices at store b satisfy

$$q_{xb} + \sum_{j' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b, \quad (1)$$

where $c_{x'b}$ is the marginal cost of supplying product x' to final consumers through retailer b , including the manufacturing and shipping costs incurred by the producer of x' and the retailing costs incurred by b .¹²

Operating profits: Equation (1) implies the standard result that the within-retailer cannibalization effect exactly offsets the cross-store substitution effect, so the mark-up rule is simply (Atkeson and Burstein, 2008; Hottman *et al.*, forthcoming; Bernard and Dhingra, 2015):

$$\frac{p_{xb} - c_{xb}}{p_{xb}} = \frac{1}{\eta}.$$

And since each retailer perceives the elasticity of demand for each of the products it offers to be η , the instantaneous profit flow jointly generated by retailer b and its suppliers is¹³

That is, consumers first allocate spending across product categories, then across retailers in each category. This formulation is used in Atkin, et al (2015). Which specification is preferable depends upon the importance of transport and shopping time costs to consumers.

¹²Note that here since buyer-seller pairs set retail prices to maximize the value of the surplus generated by their business relationships, c_{xb} is the cost for the pair to offer the product in retail market, which can include seller acquisition cost, transportation cost, and retailer inventory cost.

¹³See appendix A for details.

$$\pi_b^T = \frac{E}{\eta P^{1-\eta}} \left[\sum_{x \in J_b} \left(\frac{\eta}{\eta-1} \right)^{1-\alpha} \tilde{c}_x^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_b^{\eta-1}, \quad (2)$$

where $\tilde{c}_x = \frac{c_{xb}}{\xi_{xb}}$ is the quality-adjusted marginal cost incurred by buyer-seller pair $x - b$ per unit supplied in the retail market.

3.2 The Wholesale Market and Payoff Functions

Buyer-seller transfers: We can now describe the flow pay-off functions for buyers (retailers) and sellers (foreign exporters) in the wholesale market. Suppose there are I intrinsic buyer types indexed by $i \in \{1, 2, \dots, I\}$, so that if buyer b is a type- i retailer, $\mu_b = \mu_i$. Similarly, suppose there are J intrinsic seller types indexed by $j \in \{1, 2, \dots, J\}$, so that if seller x is type- j exporter, matches between this seller and a type- i buyer generate a quality-adjusted marginal cost of \tilde{c}_j . Finally, let $\mathbf{s} = \{s_1, s_2, \dots, s_J\}$ be a vector of counts of the number of sellers of each type currently matched to a particular buyer. Then by equation (2), the gross profit flow accruing to a type- i buyer and its portfolio of suppliers \mathbf{s} is:

$$\pi_i^T(\mathbf{s}) = \frac{E}{\eta P^{1-\eta}} \left[\sum_j \left(\frac{\eta}{\eta-1} \right)^{1-\alpha} s_j \tilde{c}_j^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_i^{\eta-1} \quad (3)$$

Note that when the elasticity of substitution across retailers exceeds the elasticity of substitution across products ($\alpha > \eta > 1$), this surplus exhibits diminishing returns with respect to the number of suppliers of any type. That is, buyers who add additional sellers reduce total surplus per supplier.

To determine the division of this profit flow between a particular buyer and her portfolio of sellers, we assume that the total surplus associated with a particular buyer-seller match is

divided up according to the Stole and Zwiebel (1996) bargaining protocol.¹⁴ As demonstrated in the appendix, this implies that at each point in time the profit flow transferred to each type j seller is

$$\begin{aligned}\tau_{ji}(\mathbf{s}) &\approx \beta \frac{\partial \pi_i^T(\mathbf{s})}{\partial s_j} \\ &= \frac{\beta}{\alpha - 1} \left(\frac{\eta}{\eta - 1} \right)^{-\eta} \frac{E}{P^{1-\eta}} \left[\sum_j s_j \tilde{c}_j^{1-\alpha} \right]^{\frac{\alpha-\eta}{1-\alpha}} \tilde{c}_j^{1-\alpha} \mu_i^{\eta-1}\end{aligned}\tag{4}$$

where $\beta \in [0, 1]$ is a parameter measuring the bargaining strength of the seller, and the equality is approximate because we have used a derivative to describe a discrete one-unit change in s_j .

Expressing the transfer function in observables: Equation (4) provides a basis for estimating some key parameters of our model, but several transformations are necessary in order to bring it to the data. First, since $\tau_{ji}(\mathbf{s})$ is not observable, we need to convert it to an expression describing the flow of export payments from a type- i buyer to a type- j seller in state \mathbf{s} . Recognizing that exports payments include both exporter profits, $\tau_{ji}(\mathbf{s})$, and compensation for the exporter's production costs, this is straightforward. As shown in the appendix, if some fraction λ of the variable costs $c_j q_{ji}$ incurred by an $i - j$ partnership is attributable to the seller, her flow export revenues from the partnership are:

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}} \left(\frac{\eta}{\eta - 1} \right)^{-\eta} \left[\sum_{\ell=1}^J s_{\ell} \tilde{c}_{\ell}^{1-\alpha} \right]^{\frac{\alpha-\eta}{1-\alpha}} \tilde{c}_j^{1-\alpha} \mu_i^{\eta-1} \left[\frac{\beta}{\alpha - 1} + \lambda \left(\frac{\eta}{\eta - 1} \right)^{\eta-1} \right].\tag{5}$$

Second, neither quality-adjusted marginal costs, \tilde{c}_{ℓ} , nor counts of the different types of sellers, s_{ℓ} , are observable. However, we can eliminate the sum in square brackets by using the

¹⁴Under the Stole and Zwiebel (1996) bargaining protocol, buyers bargaining continuously with each of the sellers they are matched with, treating each as the marginal supplier.

within buyer i revenue share of a type- j seller:

$$h_{j|i} = \frac{\tilde{c}_j^{1-\alpha}}{\sum_{\ell=1}^J s_\ell \tilde{c}_\ell^{1-\alpha}} \quad (6)$$

Thus we can rewrite equation (5) in terms of observables and fixed effects:

$$r_{ji}(\mathbf{s}) = (h_{j|i})^{\frac{\alpha-\eta}{\alpha-1}} \frac{E}{P^{1-\eta}} \left(\frac{\eta}{\eta-1} \right)^{-\eta} \left(\frac{\mu_i}{\tilde{c}_j} \right)^{\eta-1} \left[\frac{\beta}{\alpha-1} + \lambda \left(\frac{\eta}{\eta-1} \right)^{\eta-1} \right] \quad (7)$$

An even simpler expression obtains in the special case where cost per unit quality does not vary across products within retailers: $\tilde{c}_j = \tilde{c}$. Then equation (5) collapses to

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}} \left(\frac{\eta}{\eta-1} \right)^{-\eta} s^{\frac{\alpha-\eta}{1-\alpha}} \left(\frac{\mu_i}{\tilde{c}} \right)^{\eta-1} \left[\frac{\beta}{\alpha-1} + \lambda \left(\frac{\eta}{\eta-1} \right)^{\eta-1} \right] \quad (8)$$

where $s = \sum_{\ell=1}^J s_\ell$ is the total number of sellers matched to the buyer, an observable variable.

3.3 Search and Matching

3.3.1 Market aggregates and Market Slackness

Next we characterize matching patterns in wholesale markets. For expositional clarity, we focus on the case of a single type of seller, and thereby reduce the vector \mathbf{s} to the scalar, s . The more general case of multiple seller types is treated in our appendix.

First, we introduce variables that measure agents' "visibility." The key feature of these objects is that, for any two agents or groups of agents on the same side of the market, the ratio of their visibilities is also the ratio of their hazards for meeting a new business partner.

Let $M_i^B(s)$ be the measure of type- i buyers with s sellers, and define these buyers' visibility to be:

$$H_i^B(s) = \sigma_i^B(s) M_i^B(s)$$

where $\sigma_i^B(s)$ measures the search intensity of any one of these buyers. Aggregating over types and partner counts, the overall visibility of buyers is measured by:

$$H^B = \sum_{i=1}^I \sum_{s=0}^{s_{\max}} H_i^B(s)$$

Analogously, let $M_j^S(n)$ be the measure of type j sellers with n buyers, and suppose each of these sellers searches with intensity $\sigma_j^B(n)$. Then this group's visibility is measured by:

$$H_j^S(n) = \sigma_j^S(n)M_j^S(n),$$

and the overall visibility of sellers to buyers is:

$$H^S = \sum_{j=1}^J \sum_{n=0}^{n_{\max}} H_j^S(n)$$

Following much of the labor search literature, we assume a matching function that is homogeneous of degree one in the visibility of buyers and sellers. Specifically we assume that the measure of matches per unit time is given by (Petrongolo and Pissarides, 2001):¹⁵

$$X = f(H^S, H^B) = H^B \left[1 - \left(1 - \frac{1}{H^B} \right)^{H^S} \right] \approx H^B \left[1 - e^{-H^S/H^B} \right] \quad (9)$$

From buyers' perspective, we can then define market slackness in a manner analogous to random search models:

$$\theta^B = \frac{f(H^S, H^B)}{H^B}.$$

The larger is θ^B , the more matches take place for a given amount of buyer visibility. Likewise, market slackness from sellers' perspective is:

$$\theta^S = \frac{f(H^S, H^B)}{H^S}. \quad (10)$$

¹⁵Other matching functions are of course feasible here. We have also experimented with $x = \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$.

Finally, assuming random matching, the share of matches involving buyers of type i with $s > 0$ sellers is:

$$\frac{\sigma_i^B(s)M_i^B(s)}{H^B} \quad (11)$$

and the share of matches involving sellers of type j with $n > 0$ buyers is:

$$\frac{\sigma_j^S(n)M_j^S(n)}{H^S}.$$

In the absence of \tilde{c} heterogeneity across seller types, sellers' payoffs do not depend upon j . And if sellers' search cost functions do not depend upon their type either, we can drop the j subscript from $\sigma_j^S(n)$. For the time being we do so.

3.3.2 Optimal search

It remains to characterize the policy functions $\sigma_i^B(s)$ and $\sigma^S(n)$ that maximize the values of agents' expected payoff streams. To do this we introduce buyer and seller search cost functions, which measure the flow cost of sustaining search intensities σ^B and σ^S , respectively:

$$\begin{aligned} k^B(\sigma^B, s) &= \frac{(\sigma^B)^{\nu_B}}{(s+1)^{\gamma^B}} \\ k^S(\sigma^S, n) &= \frac{(\sigma^S)^{\nu_S}}{(n+1)^{\gamma^S}} \end{aligned}$$

By assumption, search costs are positive and convex in search intensity: $\nu_B, \nu_S > 1$. Also, network effects may reduce the costs of forming new matches as agents' partner counts grow:

$$\gamma^B, \gamma^S \geq 0.$$

Buyer's problem: Given that type- i buyers enjoy profit flow $\pi_i^B(s)$ when they are matched with s suppliers, such buyers choose their search intensity to solve:

$$V_i^B(s) = \max_{\sigma^B} \left\{ \frac{\pi_i^B(s) - k^B(\sigma^B) + s\delta V_i^B(s-1) + \sigma^B \theta^B V_i^B(s+1)}{\rho + s\delta + \sigma^B \theta^B} \right\} \quad (12)$$

where ρ is the rate of time preference and $V_i^B(s)$ is the present value of a type- i buyer that is currently matched with s sellers. Intuitively, the seller reaps profit flow $\pi_i^B(s) - k^B(\sigma^B)$ until the next event occurs. With hazard $s\delta$ this event is an exogenous termination of one of the s relationships, and with hazard $\sigma^B \theta^B$ it is a new match.

The optimal search policy for type- i buyers with s sellers, $\sigma_i^B(s)$, therefore satisfies

$$\frac{\partial k^B(\sigma^B, s)}{\partial \sigma^B} = \theta^B [V_i^B(s+1) - V_i^B(s)]. \quad (13)$$

Sellers' problem: Since sellers have constant marginal costs, the number of buyers they currently supply does not affect their expected returns from adding another one. On the other hand, the seller's payoff function from a particular match, $\tau_i(s)$, depends upon the buyer's type, i , and the buyer's current seller count, s , so ex post, it matters whom sellers match with. The value to any seller of matching with a type- i buyer who has s suppliers is:¹⁶

$$V_{i,s}^S = \frac{\tau_i(s) + (s-1)\delta V_{i,s-1}^S + \sigma_i^B(s)\theta^B V_{i,s+1}^S}{\rho + s\delta + \sigma_i^B(s)\theta^B}. \quad (14)$$

Intuitively, a business relationship with a type- i buyer who has s suppliers will terminate with exogenous hazard δ , become a relationship with a type- i buyer who has $s-1$ suppliers with hazard $(s-1)\delta$. Analogously, it will become a relationship with a type- i buyer who has $s+1$ suppliers with hazard $\sigma_i^B(s)\theta^B$.

¹⁶The destruction hazard δ is weighted by $(s-1)$ to adjust for the fact that the seller's own relationship with the buyer may die, in which case the continuation value of this relationship for this seller is zero. Of course V_s^S makes sense only if $s > 0$, as a seller can't have a connection with a buyer with zero sellers.

Taking expectations over the population of buyers that sellers might meet, the *ex ante* value of a new relationship is:

$$V^S = \sum_i \sum_{s=0}^{\infty} V_{i,s+1}^S P_i^B(s),$$

where $P_i^B(s) = H_i^B(s)/H^B$ is the relative visibility of buyers who are type- i and have s sellers. So the optimal search intensity for any seller with n buyers satisfies:

$$\frac{\partial k^S(\sigma^S, s)}{\partial \sigma^S} = \theta^S V^S. \quad (15)$$

3.3.3 Equilibria and Transition Dynamics

Equations of motion: Given that all relationships end with exogenous hazard δ , the equation of motion for the measure of buyers of type i with s sellers is:

$$\begin{aligned} \dot{M}_i^B(s) &= \sigma_i^B(s-1)\theta^B M_i^B(s-1) + \delta(s+1)M_i^B(s+1) \\ &\quad - (\sigma_i^B(s)\theta^B M_i^B(s) + \delta s M_i^B(s)). \\ s &= 1, \dots, s_{\max}; \quad i = 1, \dots, I \end{aligned} \quad (16)$$

This group gains a member whenever any of the $M_i^B(s-1)$ buyers with $s-1$ suppliers adds a supplier, and the hazard of this happening is $\sigma_i^B(s-1)\theta^B$. Similarly, it gains a member whenever any of the $M_i^B(s+1)$ buyers with $s+1$ suppliers loses a supplier because of exogenous attrition, and this occurs with hazard $\delta(s+1)$. By analogous logic, the group loses existing members that either successfully add a supplier (with hazard $\sigma_i^B(s)\theta^B$) or loses one (with hazard δ). Finally, the measure of buyers of type i with $s=0$ sellers evolves according to:

$$\dot{M}_i^B(0) = \delta M_i^B(1) - \sigma_i^B(0)\theta^B M_i^B(0) \quad i = 1, \dots, I \quad (17)$$

Replacing B with S and s with n in (16) and (17), the equations of motion for seller measures $M_j^S(n)$ obtain.

Steady state: To characterize the steady state of this system, we set $\dot{M}_i^B(s) = \dot{M}_j^S(n) = 0$ and solve the system of $I \cdot (s_{\max} + 1) + J \cdot (n_{\max} + 1)$ equations implied by both versions of (16) and (17)—for buyers and sellers. In doing so we, treat the measures of each type of intrinsic type as exogenous constants and impose the adding-up constraints:

$$M_i^B = \sum_{s=0}^{s_{\max}} M_i^B(s) \quad (18)$$

$$M_j^S = \sum_{n=0}^{n_{\max}} M_j^S(n), \quad (19)$$

Transition dynamics: Solving for transition dynamics is more involved. Suppose we wish to find the transition path from one market environment to a new one under perfect foresight. We begin by finding the steady distribution of buyers and sellers across types for the new regime, as well as the associated value functions. We then guess the trajectory of endogenous market-wide aggregates $\{\theta^B(t), \theta^S(t), P(t)\}$ from the initial state to this steady state, and solve for buyer and seller distribution functions using backward induction and finite differencing. Appendix C provides details.

3.4 Introducing assortative matching

Thus far, our model does not allow for the possibility that some retailers specialize in athletic shoes, while others are more about dress shoes, and still others do both types of business. Nor it does it provide a mechanism through which assortative matching on the basis of product quality might be accommodated. These features of the model can be relaxed by introducing a

compatibility function. This exercise is tangential to our purposes, so we relegate details of this extension to the appendix.

4 Fitting the model to data

In this section, we calibrate the model to the data and assess the quality of the fit.

4.1 Transfer function estimates

Our data allow us to calculate annual payments from each Colombian footwear importer to each of its foreign suppliers. These bilateral payment records provide a means for estimating equation (7), which we re-state here in log form, adding time dummies d_t and a stochastic match-specific shock $\varepsilon_{j,i}$:

$$\ln r_{jit} = \left(\frac{\alpha - \eta}{\alpha - 1} \right) \ln h_{j|i,t} + \ln \left(\frac{\mu_i}{\tilde{c}_j} \right)^{\eta-1} + d_t + \varepsilon_{jit} \quad (20)$$

The time dummies d_t are meant to capture the constant $\ln \left\{ \left(\frac{\eta}{\eta-1} \right)^{-\eta} \left[\frac{\beta}{\alpha-1} + \lambda \right] \right\}$ and variation in $\ln \frac{E}{P^{1-\eta}}$ over time.

In estimating this equation, we face several choices. First, we must decide how to handle the term $\ln \left(\frac{\mu_i}{\tilde{c}_j} \right)^{\eta-1}$. One option is to absorb it with match-specific fixed effects; an alternative is to impose that buyer and seller effects on marginal costs are log-separable, so that separate buyer and seller fixed effects suffice. Second, we must decide whether to treat $\ln h_{j|i,t}$ as exogenous, and if not, what instrumenting strategy to use. Under the assumptions of the model, all variation in $h_{j|i}$ is driven by random matching patterns and no instruments are needed. However, to the extent that the data reflect covariation in $h_{j|i,t}$ and r_{jit} due to

	OLS-FE		IV-FE		OLS-FE	
	rubber	textiles	rubber	textiles	rubber	textiles
$\ln h_{i j}$	0.823 (0.019)	0.818 (0.019)	0.669 (0.062)	0.969 (0.071)	–	–
$\ln n_i$	–	–	–	–	–0.382 (0.088)	–0.289 (0.095)
match effects	yes	yes	yes	yes	no	no
buyer effects	no	no	no	no	yes	yes
year effects	yes	yes	yes	yes	yes	yes
R ²	0.937	0.855	0.326	0.291	0.425	0.454
obs.	3,445	3,245	2,859	2,775	3,445	3,245

Table 6: Estimation of transfer equation

transitory demand or marginal cost shocks, we expect an upward bias in our estimate of $\left(\frac{\alpha-\eta}{\alpha-1}\right)$ unless some type of IV estimator is used. Even without transitory shocks, we might prefer to use an IV approach because of measurement error. For example, if sellers' shares in marginal costs (λ) were to vary across matches, then equation (6) would only hold approximately.

Table 6 reports estimates of the transfer equation for two of the larger footwear categories—rubber uppers and textile uppers. (Results for the leather uppers are similar; we leave them out to conserve space.) The first two columns are obtained by applying an OLS fixed effects to equation (20). The results imply that there are diminishing returns to adding additional sellers of any type, or put differently, the elasticity of substitution across varieties within a store (α) exceeds the elasticity of substitution across stores (η).

The next two columns report estimates of the same equation, except $h_{j|i,t}$ is treated as correlated with the error term and an IV fixed effects estimator is used. Here the instrument is a share-weighted average of number of buyers of the *other* sellers at buyer j , which should be correlated with $h_{j|i,t}$ to the extent that cost or product appeal shocks specific to these sellers will affect the revenue share of seller j . (Of course, this instrument is not motivated by the

model, and in that sense is less than ideal.) The IV estimates of $\left(\frac{\alpha-\eta}{\alpha-1}\right)$ are not systematically different from those in the first two columns.

The last two columns report OLS fixed effects estimates of (8), and thus embody the assumption that $\tilde{c}_j = \tilde{c}$. The explanatory variable is now the log of the total number of sellers, s , and the coefficient on this variable is $\frac{\alpha-\eta}{1-\alpha} = -\frac{\alpha-\eta}{\alpha-1}$. Recognizing this sign flip, we note that the estimates are qualitatively consistent with the share-based estimates, albeit smaller in absolute value. We interpret this difference in magnitudes as attenuation bias, since any amount of seller heterogeneity will make $\ln s$ a noisy approximation to the conceptually appropriate variable, $h_{j|i,t}$.

4.2 A preliminary calibration

We now move to a preliminary calibration of the dynamic structural model.¹⁷ To keep the calculations simple, we shut down seller heterogeneity and assume that $\tilde{c}_j = \tilde{c}$. Consistency then dictates that we use the estimates of $\frac{\alpha-\eta}{\alpha-1}$ and $var[(\eta-1)\ln\mu]$ that obtain when $var(\tilde{c}_j) = 0$ is imposed, i.e., those reported in the last two columns of Table 6.

Conditioning on these estimates, we proceed to assign values to the elasticity of substitution across products, α , the dispersion in the buyer types, $var(\mu)$, the search cost parameters $(k_0^B, k_0^S, \nu_V, \nu_S, \gamma^B, \gamma^S)$, the exogenous separation hazard, δ , and the discount rate, ρ . Some of these we fix *ex ante*. First, based on estimates in Hottman *et al.* (forthcoming), we set $\alpha = 4.35$. Next, following the macro literature, we assume a discount rate of $\rho = 0.05$. Finally, we impose symmetry across buyers and sellers in the search cost scalars, $k_0^B = k_0^S = k_0$, and

¹⁷A more careful estimation that allows for seller heterogeneity and exploits a much larger set of moments is in progress.

	6402	6403	6404		
# sellers	rubber	leather	textile	Overall	Model
1	0.453	0.496	0.469	0.489	0.475
2	0.163	0.159	0.149	0.160	0.178
3	0.073	0.080	0.100	0.074	0.089
4	0.069	0.064	0.072	0.052	0.053
5	0.060	0.053	0.051	0.050	0.036
6 -10	0.130	0.119	0.122	0.118	0.078
10+	0.052	0.029	0.037	0.058	0.090

Table 7: Estimated Degree Distribution

we simply assume that both cost functions are quadratic in search intensity: $\nu_V = \nu_S = 2$.

Given our assumption that $\alpha = 4.35$, we can infer from $\frac{\alpha-\eta}{\alpha-1} \approx 0.3$ that $\eta \approx 3.35$. Also, since we estimate $var[(\eta - 1) \ln \mu] \approx 2.2$ from the OLS-FE regressions in columns 5 and 6 of Table 6, we calculate $var(\ln \mu) = 2.2/(3.35 - 1)^2 = 0.40$, and we discretize the associated distribution of buyer effects using the method suggested in Kennan (2006)

It remains to discuss the search cost level parameter k_0 , the network effects in search cost γ^B and γ^S , and the match death hazard δ . To calibrate these parameters we minimize the sum of differences between the model and 2006 - 2009 data in the degree distribution of sellers per buyer and buyers per seller as well as the first several columns of the partner transition matrix. Loosely speaking, the degree distributions pin down the search cost parameters, and the transition matrix identifies the match death hazard.

This exercise yields a search cost level parameter $k_0 = 0.9$, and more interestingly, network effects of $\gamma^S = 0.65$ and $\gamma^B = 0.7$. These parameters imply that network effects play an important role in the model's ability to match the data. In particular, as in Eaton *et al.* (2014), reductions in search costs due to high visibility allow the model to explain the very large firms that populate the right-hand tails of the client distributions (Figures 1 and 2).

	Data		Model	
	1	2	1	2
1	0.75	0.17	0.70	0.16
2	0.42	0.34	0.49	0.32
3	0.18	0.24	0.32	0.35
4	0.13	0.15	0.19	0.32
5	0.04	0.07	0.11	0.25
6	0.02	0.07	0.07	0.18
7	0.00	0.00	0.04	0.12
8	0.00	0.00	0.02	0.08
9	0.00	0.00	0.01	0.05
10	0.00	0.06	0.00	0.03

Table 8: Estimated Transition Matrix

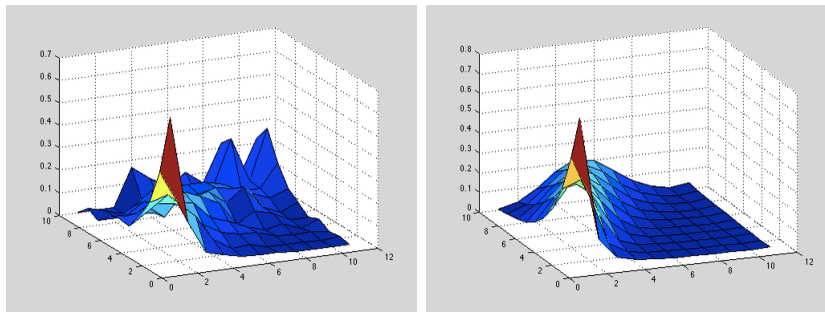


Figure 5: Transitions, sellers per buyer, data vs model

The calibration also generates a match death hazard of $\delta = 0.8$, implying an average duration of 1.25 years.

Aside from underestimating the number of buyers with one seller, the model replicates the data-based client distributions quite well (Table 7). It also captures the shape of the partner transition matrix (Figure 5), including the general tendency to lose clients over time. However, with only four free parameters, it fails to replicate the spikes that occur at high s values.

5 Putting the model to work

5.1 preliminary counterfactuals

In this section we run counterfactual experiments with the model.

Our first experiment is to reduce the search cost parameter k_0 by 30 percent, scaling back costs proportionately at all levels of search intensity. Roughly speaking, we think of this exercise as approximating improvements in global communications, and perhaps also the effects of better access to intermediaries in Panama.¹⁸ Again, we see that the tail of the degree distribution of sellers per buyer gets fatter (Figure 7).

For this experiment, we show the full transition from the estimated steady state to eight years after the shock (Figure 6). One implication is that it takes 6-7 years for the welfare benefits of lower costs to be realized. These amount to more than 10 percent per year. Here the gains are driven partly by the increase in the number of varieties available to consumers, and partly by the fact that varieties are spread across more retailers.

5.2 Interpreting the value functions

Exploiting the structure of our model, we can measure the intangible capital stocks that retailers accumulate as they build international business relationships. Figure 8 depicts $V_i^B(s)$ values for the different intrinsic buyer types, and shows how these values vary as firms add and lose clients. To reduce clutter, here we have averaged the value functions across the firms with μ_i values in the lowest tercile (denoted "low μ "), the middle tercile (denoted "medium

¹⁸**Note:** this experiment will be replaced by an exercise in which the reduction in k_0 is chosen to replicate the growth of trade flows, conditioned on the observed increase in the number of exporters.

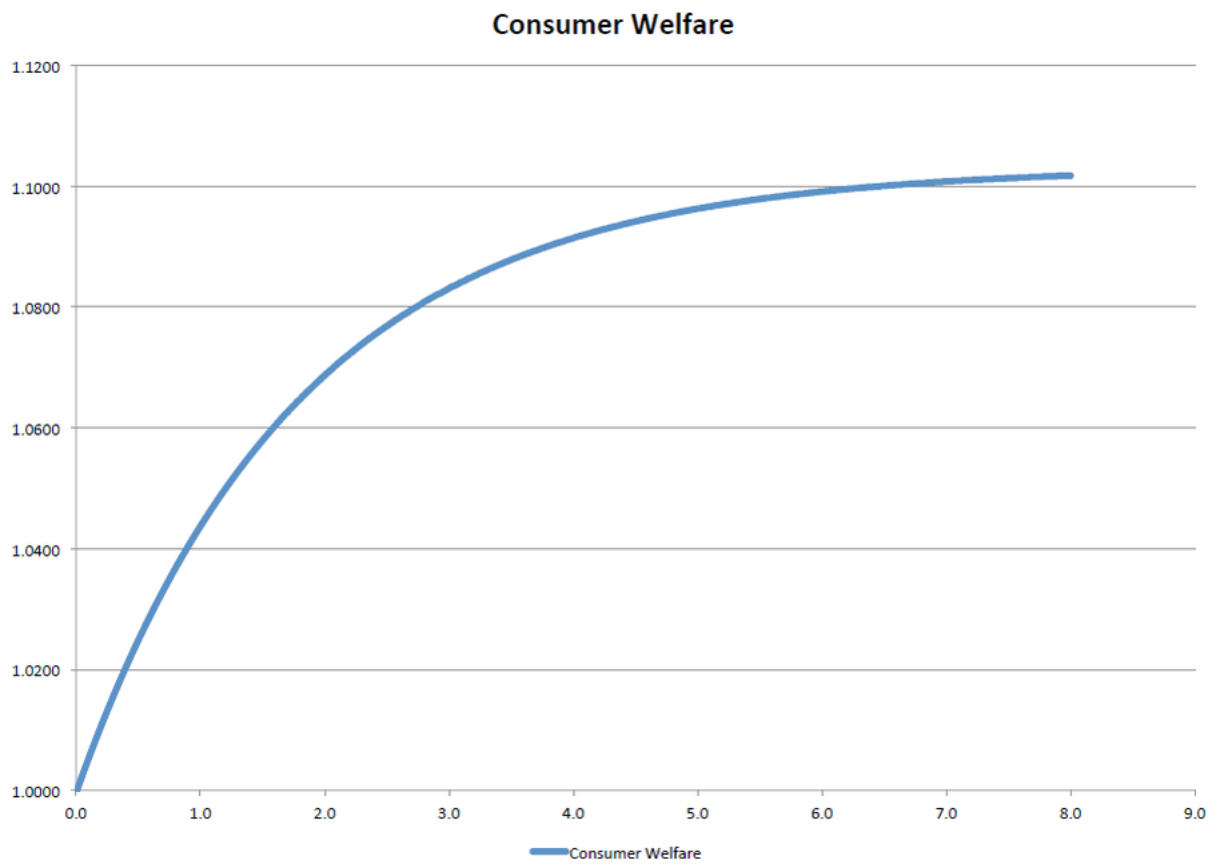


Figure 6: 30% reduction in search cost: Welfare

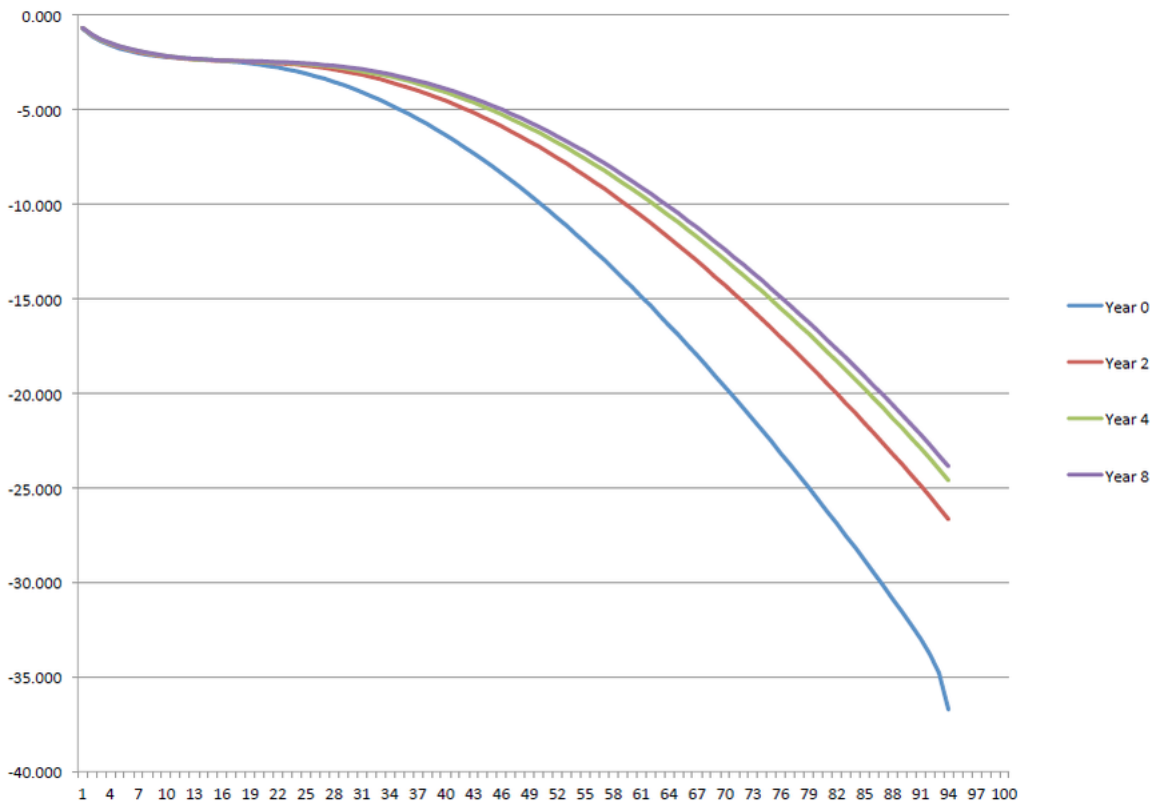


Figure 7: 30% reduction in search cost: Sellers per buyer

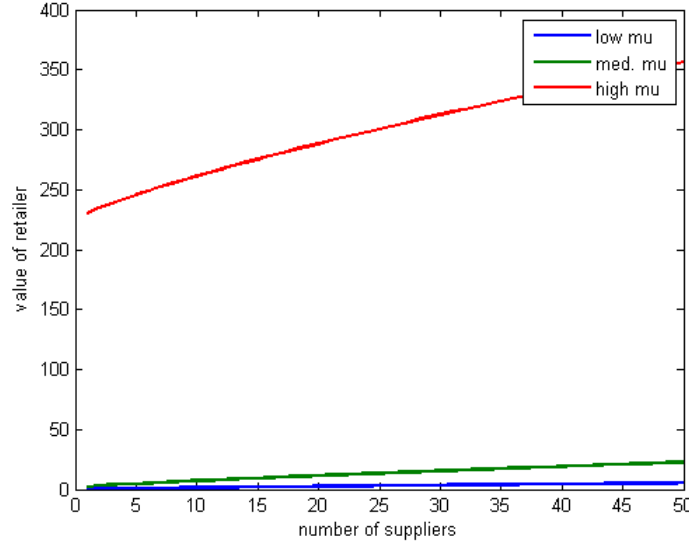


Figure 8: Retail firm values by type and state

μ "), the upper tercile (denoted "high μ "). All values are normalized by the cross-importer average annual value of imports, US\$ 7,665.

Several features of this figure merit note. First, $V_i^B(s)$ increases with the number of clients because each client adds value to the retailer by increasing the flow of rents. This is especially true at high- μ retailers, where relatively large sales volumes are generated per variety sold. Second, however, because of diminishing returns to varieties ($(\frac{\eta-1}{\alpha-1}) < 1$) and convex search costs, $V_i^B(s)$ is concave in s . Finally, the international business connections of high- μ shoe retailers are quite valuable. Consider, for example, a high- μ retailer with 30 suppliers. If we took these suppliers away but permitted it to search for replacements, its value would drop by $(300 - 230) \times \$7,665 \approx \$540,000$. Or, if we took these connections away and did not let it replace them, its value would drop by $300 \times \$7,665 \approx \$2,300,000$.

Whatever portfolio of sellers a retailer happens to have, the value of its international business relationships is sensitive to market conditions. Returning to our counterfactual ex-

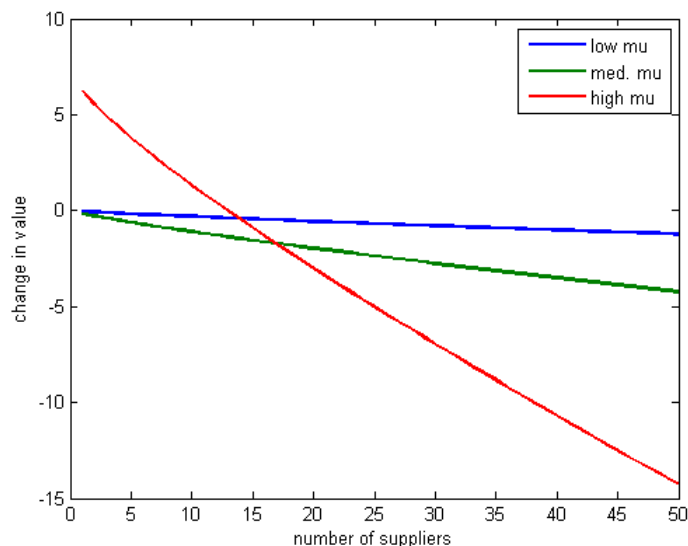


Figure 9: Capital gains for retailers with search cost reduction

periments, we now ask how they adjust as search costs fall. Figure 9 depicts the change in value for each of our three classes of retailers as a function of the number of suppliers they have.

Two forces are in play in this graph. First, reductions in international search costs generate capital losses because the value of any business relationship is bounded by the costs of replacing it. Firms that have invested in building an extensive portfolio of foreign suppliers therefore lose more value than firms of the same intrinsic appeal that have not. Second, however, reductions in search costs make it less costly for firms to expand, and this is particularly important to high- μ firms that currently have just a few business partners. These firms are net beneficiaries of search cost reductions. Put differently, high-quality start-ups prefer a world with low search frictions, while established retailers would rather see their investments in suppliers maintain their values.

6 Summary

We have developed a dynamic model of international buyer-seller matching in which search intensities are optimally chosen on both sides of the markets, and we have shown that it nicely captures key cross-sectional and dynamic features of international business relationships. Counterfactual exercises based on the model yield several basic messages. First, changes in the population of foreign suppliers—especially in China—led to substantial welfare improvements among Colombian consumers. Second, reductions in search frictions also have the potential to generate large welfare gains. Third, however, search frictions spread firms’ adjustments to market shocks over substantial periods, so that the full benefits of greater market participation by foreign suppliers may take 8-10 years to accrue. Finally, because of these search frictions, connections with foreign business partners are an important component of retailers’ intangible capital stock. For the largest retailers, these can be worth millions of dollars.

The empirical application we report is preliminary. In future drafts we plan to incorporate seller-side heterogeneity and to exploit a larger set of moments in the estimation exercise. We also hope to explore applications to other markets, including the U.S. market for apparel.

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Appendix

A Demand and Pricing

Using standard CES results, we begin by characterizing prices and market shares for a particular retailer b offering a particular subset of product varieties in the group, $x \in J_b$:

$$C_b = \left(\sum_{x \in J_b} (\xi_x C_b^j)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}}, \quad C = \left(\int_b (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (\text{A-1})$$

$$P_b = \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x} \right)^{1-\alpha} \right]^{1/(1-\alpha)}, \quad P = \left[\int_b \left(\frac{P_b}{\mu_b} \right)^{(1-\eta)} \right]^{1/(1-\eta)} \quad (\text{A-2})$$

$$h_b = \frac{\left(\frac{P_b}{\mu_b} \right)^{1-\eta}}{P^{1-\eta}}, \quad h_{x|b} = \frac{\left(\frac{P_{xb}}{\xi_x} \right)^{1-\alpha}}{P_b^{1-\alpha}}$$

These expressions imply the revenue generated by retail sales of product x at store b is:

$$\begin{aligned} R_{xb} &= P_{xb} q_{xb} \\ &= h_{x|b} h_b E \\ &= \mu_b^{\eta-1} \xi_x^{\alpha-1} P_{xb}^{1-\alpha} P_b^{\alpha-\eta} P^{\eta-1} E \end{aligned} \quad (\text{A-3})$$

Since we assume a continuum of buyers, $\frac{\partial \ln P}{\partial \ln P_b} = 0$. Also,

$$\begin{aligned} \frac{\partial \ln P_b}{\partial \ln P_{ib}} &= \frac{\partial P_b}{\partial P_{ib}} \frac{P_{ib}}{P_b} \\ &= 1/(1-\alpha) \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x} \right)^{1-\alpha} \right]^{1/(1-\alpha)-1-1/(1-\alpha)} \left[(1-\alpha) \left(\frac{P_{ib}}{\xi_i} \right)^{1-\alpha} \right] \\ &= \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x} \right)^{1-\alpha} \right]^{-1} \left[\left(\frac{P_{ib}}{\xi_i} \right)^{1-\alpha} \right] = h_{i|b} \end{aligned}$$

Bertrand-Nash pricing therefore implies:

$$\begin{aligned}\frac{\partial \ln R_{xb}}{\partial \ln P_{xb}} &= (1 - \alpha) + h_{x|b}(\alpha - \eta) \\ \frac{\partial \ln R_{xb}}{\partial \ln P_{x'b}} &= h_{x'|b}(\alpha - \eta) \quad \forall x' \neq x\end{aligned}$$

Plugging these expressions into the first-order conditions for pricing,

$$q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b,$$

we obtain:

$$\begin{aligned}\frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{E} \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + \sum_{x' \in J_b, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{x'b}}{E} \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0 \\ \frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{1}{q_{xb}} \left(\frac{p_{xb} q_{xb}}{E} \right) \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + \sum_{x' \in J_b, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{1}{q_{x'b}} \left(\frac{q_{x'b} p_{x'b}}{E} \right) \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0 \\ \frac{p_{xb} q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{q_{xb}} \left(\frac{p_{xb} q_{xb}}{E} \right) \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + \sum_{x' \in J_b, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{x'b}}{q_{x'b}} \left(\frac{q_{x'b} p_{x'b}}{E} \right) \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0 \\ h_{xb} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{q_{xb}} (h_{xb}) \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + \sum_{x' \in J_b, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{x'b}}{q_{x'b}} h_{j|b} \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0 \\ h_{xb} + (-\alpha + (\alpha - \eta) h_{x|b}) (h_{xb}) \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + \sum_{x' \in J_b, x' \neq x} ((\alpha - \eta) h_{x'|b}) h_{xb} \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0 \\ 1 - \alpha \left(\frac{p_{xb} - c_{xb}}{p_{xb}} \right) + (\alpha - \eta) \sum_{x' \in J_b} h_{x'|b} \left(\frac{p_{x'b} - c_{x'b}}{p_{x'b}} \right) &= 0\end{aligned}$$

where $h_{xb} = h_{x|b} h_b = \frac{p_{xb} q_{xb}}{E}$. From this we can infer $\epsilon_{xx}^b = \frac{\partial \ln h_{xb}}{\partial \ln P_{xb}} - 1 = -\alpha + h_{x|b}(\alpha - \eta)$, and

by analogous logic, $\epsilon_{xx'}^b = (\alpha - \eta) h_{x|b}$.

Next, plugging these expressions into the first-order conditions for pricing, we obtain:

$$\begin{aligned}q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \quad \forall j \in J_b, \\ q_{xb} + \sum_{x' \in J_b} \epsilon_{xx'}^b \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \\ q_{xb} + \sum_{\substack{x' \neq x \\ x' \in J_b}} [h_{x|b}(\alpha - \eta)] \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) + [(\alpha - \eta) h_{x|b}] \frac{q_{xb}}{p_{xb}} (p_{xb} - c_{xb}) &= 0\end{aligned}$$

Since this relationship holds for all $x \in J_b$, the mark-up for each product must be the same.

Call it $m = \frac{p_{xb} - c_{xb}}{p_{xb}}$ and reduce this equation to $1 - \alpha m + (\alpha - \eta)m = 0$, or

$$m = \frac{1}{\eta}.$$

Essentially the same result can be found in Atkeson and Burstein (2008) and Hottman *et al.* (forthcoming).

B Value functions with heterogenous buyers

Let $\mathbf{s} = \{s_1, s_2, \dots, s_J\}$ be a vector of counts of the number of sellers of each type $j \in \{1, 2, \dots, J\}$ who are attached to a particular buyer, and let $\mathbf{s}_{-j} = \{s_1, s_2, \dots, s_{j-1}, s_{j+1}, \dots, s_J\}$ be the same vector without its j^{th} element, so that (s_j, \mathbf{s}_{-j}) is one way to indicate that a seller is in state \mathbf{s} .

The buyer-to-seller transfer function $\tau_{ji}^S(\mathbf{s})$ and the type- i buyer payoff function $\pi_i^B(\mathbf{s})$ are chosen to satisfy the surplus sharing rule

$$(1-\beta)V_i^S(s_j, \mathbf{s}_{j-1}) = \beta [V_i^B(s_j, \mathbf{s}_{-j}) - V_i^B(s_j - 1, \mathbf{s}_{-j})], \quad s_j \in \{1, 2, \dots, s^{\max}\}, \quad i \in \{1, 2, \dots, N^B\}, \quad (\text{A-4})$$

We now derive closed-form expressions for the surplus shares implied by (A-4). The logic is similar to that found in Bertola and Garibaldi (2001), though it is adapted to our discrete state space.

Suppressing buyer-type indices, the flow value of a type- i buyer who is currently in state

\mathbf{s} is:

$$\begin{aligned} \rho V_i^B(\mathbf{s}) &= \pi_i^B(\mathbf{s}) - k_s^B(\sigma_i^B(\mathbf{s})) + \sigma_i^B(\mathbf{s}) \sum_j^J \theta_j^B [V_i^B(s_j + 1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})] \\ &\quad + \delta \sum_j^J s_j [V_i^B(s_j - 1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})] \end{aligned} \quad (\text{A-5})$$

Likewise the value to a type- j seller of being matched with a type- i buyer in state \mathbf{s} is:

$$\begin{aligned} \rho V_{ji}^S(\mathbf{s}) &= \tau_{ji}(\mathbf{s}) + \sigma_i^B(\mathbf{s}) \sum_k^J \theta_k^B [V_{ji}^S(s_k + 1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})] \\ &\quad + \delta \sum_{k=1}^J (s_k - \mathbf{1}_{k=j}) [V_{ji}^S(s_k - 1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})] \end{aligned} \quad (\text{A-6})$$

Finally, the ex ante expected value of a new business relationship for a type- j seller is:

$$V_j^S = \sum_i \sum_{\mathbf{s} \in S} P_i^B(\mathbf{s}) V_{ji}^S(\mathbf{s})$$

where $P_i^B(\mathbf{s}) = H_i^B(\mathbf{s})/H^B$ is the relative visibility of type- i buyers in state \mathbf{s} .

C Bargaining

Differencing the buyer's value function and suppressing buyer type i , we have:

$$\begin{aligned} &\rho(V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})) \\ &= [\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j})] - [k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j - 1, \mathbf{s}_{-j})] \\ &\quad + \sigma^B(s_j, \mathbf{s}_{-j}) \left[\sum_{k \neq j} \theta_k^B V^B(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j + 1, \mathbf{s}_{-j}) - \theta^B V^B(s_j, \mathbf{s}_{-j}) \right] \\ &\quad - \sigma^B(s_j - 1, \mathbf{s}_{-j}) \left[\sum_{k \neq j} \theta_k^B V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j - 1, \mathbf{s}_{-j}) \right] \\ &\quad + \delta \left[\sum_{k \neq j} s_k V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) + s_j V^B(s_j - 1, \mathbf{s}_{-j}) - \bar{s} V^B(s_j, \mathbf{s}_{-j}) \right] \\ &\quad - \delta \left[\sum_{k \neq j} s_k V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k}) + (s_j - 1) V^B(s_j - 2, \mathbf{s}_{-j}) - (\bar{s} - 1) V^B(s_j - 1, \mathbf{s}_{-j}) \right] \end{aligned}$$

Now we simplify this equation in two steps. First apply a discrete approximation of the first order condition at $(s_j - 1, s_{-j})$ for the buyer search:

$$[k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j - 1, \mathbf{s}_{-j})] \approx (\sigma^B(s_j, \mathbf{s}_{-j}) - \sigma^B(s_j - 1, \mathbf{s}_{-j})) \left[\sum_{k \neq j} \theta_k^B V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j, k}) \right. \\ \left. + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j - 1, \mathbf{s}_{-j}) \right]$$

Using the above, we can simplify

$$-[k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j - 1, \mathbf{s}_{-j})] \\ + \sigma^B(s_j, \mathbf{s}_{-j}) \left[\sum_{k \neq j} \theta_k^B V^B(s_j, s_k + 1, \mathbf{s}_{-j, k}) + \theta_j^B V^B(s_j + 1, \mathbf{s}_{-j}) - \theta^B V^B(s_j, \mathbf{s}_{-j}) \right] \\ - \sigma^B(s_j - 1, \mathbf{s}_{-j}) \left[\sum_{k \neq j} \theta_k^B V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j, k}) + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j - 1, \mathbf{s}_{-j}) \right] \\ = \sigma(s_j, \mathbf{s}_{-j}) \left[\sum_{k \neq j} \theta_k^B (V^B(s_j, s_k + 1, \mathbf{s}_{-j, k}) - V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j, k})) \right. \\ \left. + \theta_j^B (V^B(s_j + 1, \mathbf{s}_{-j}) - V^B(s_j, \mathbf{s}_{-j})) - \theta^B (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})) \right]$$

Second, we can also simplify the destruction side using

$$\delta \left[\sum_{k \neq j} s_k V^B(s_j, s_k - 1, \mathbf{s}_{-j, k}) + s_j V^B(s_j - 1, \mathbf{s}_{-j}) - \bar{s} V^B(s_j, \mathbf{s}_{-j}) \right] \\ - \delta \left[\sum_{k \neq j} s_k V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j, k}) + (s_j - 1) V^B(s_j - 2, \mathbf{s}_{-j}) - (\bar{s} - 1) V^B(s_j - 1, \mathbf{s}_{-j}) \right] \\ = \delta \left[\sum_{k \neq j} s_k (V^B(s_j, s_k - 1, \mathbf{s}_{-j, k}) - V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j, k})) \right. \\ \left. + (s_j - 1) (V^B(s_j - 1, \mathbf{s}_{-j}) - V^B(s_j - 2, \mathbf{s}_{-j})) - \bar{s} (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})) \right]$$

To summarize, the above gives us:

$$\begin{aligned}
& \rho[V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})] \\
= & [\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j})] \tag{A-7} \\
& + \sigma^B(s) \left[\sum_{k \neq j} \theta_k^B (V^B(s_j, s_k + 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k})) \right. \\
& + \theta_j^B (V^B(s_j + 1, \mathbf{s}_{-j}) - V^B(s_j, \mathbf{s}_{-j})) - \theta^B (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})) \left. \right] \\
& + \delta \left[\sum_{k \neq j} s_k (V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k})) \right. \\
& \left. + (s_j - 1)(V^B(s_j - 1, \mathbf{s}_{-j}) - V^B(s_j - 2, \mathbf{s}_{-j})) - \bar{s} (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})) \right]
\end{aligned}$$

By the definition of the type j seller's value function, we have

$$\begin{aligned}
\rho V_j^S(\mathbf{s}) &= \tau^j(\mathbf{s}) + \sigma^B(s) \left[\sum_{k \neq j} \theta_k^B V_j^S(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j + 1, \mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s}) \right] \tag{A-8} \\
& \delta \left[\left(\sum_{k \neq j} s_k V_j^S(s_k - 1, \mathbf{s}_{-k}) + (s_j - 1) V_j^S(s_j - 1, \mathbf{s}_{-j}) \right) - \bar{s} V_j^S(\mathbf{s}) \right]
\end{aligned}$$

Finally, using equations (A-7), (A-8) and (A-4), we have

$$\begin{aligned}
& \beta \rho [V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})] - (1 - \beta) \rho V_j^S(s_j, \mathbf{s}_{-j}) \\
= & \beta [\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j})] + \\
& (1 - \beta) \sigma^B(s) \left[\sum_{k \neq j} \theta_k^B (V_j^S(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j + 1, \mathbf{s}_{-j})) - \theta^B V_j^S(s_j, \mathbf{s}_{-j}) \right] + \\
& \delta (1 - \beta) \left[\sum_{k \neq j} s_k (V^S(s_j, s_k - 1, \mathbf{s}_{-j,k})) + (s_j - 1)(V^S(s_j - 1, \mathbf{s}_{-j}) - \bar{s} (V^S(s_j, \mathbf{s}_{-j}))) \right] \\
& - (1 - \beta) \left[\tau^j(\mathbf{s}) + \sigma^B(s) \left[\sum_{k \neq j} \theta_k^B V_j^S(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j + 1, \mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s}) \right] \right] \\
& - (1 - \beta) \delta \left[\left(\sum_{k \neq j} s_k V_j^S(s_k - 1, \mathbf{s}_{-k}) + (s_j - 1) V_j^S(s_j - 1, \mathbf{s}_{-j}) \right) - \bar{s} V_j^S(\mathbf{s}) \right] \\
= & 0
\end{aligned}$$

Or, cancelling terms and re-arranging, the flow transfer to a type- j seller by a type- i buyer in state \mathbf{s} is share β of the total flow surplus generated by their match:

$$\tau^j(\mathbf{s}) = \beta[\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j}) + \tau^j(\mathbf{s})] \quad (\text{A-9})$$

The total flow surplus created by the marginal match between a type- j seller by a type- i buyer in state \mathbf{s} must equal the sum of the flow surpluses reaped by the buyer and the seller:

$$\pi_i^B(s_j, \mathbf{s}_{-j}) - \pi_i^B(s_j - 1, \mathbf{s}_{-j}) + \tau^j(\mathbf{s}) = \pi_i^T(\mathbf{s}) - \pi_i^T(s_j - 1, \mathbf{s}_{-j})$$

So we can re-state (A-9) as:

$$\tau_{ji}(\mathbf{s}) = \beta [\pi_i^T(\mathbf{s}) - \pi_i^T(s_j - 1, \mathbf{s}_{-j})] \quad (\text{A-10})$$

D Transition dynamics

Details to come

E Adding assortative matching

It is straightforward to modify the model so that particular sellers tend to specialize in particular types of goods. To do so, continue to assume that buyers and sellers encounter each other through an undirected search process. But now suppose that shipments only take place between *compatible* buyers and sellers who meet, and let any randomly selected pair of type- i buyer and type- j seller be compatible with probability $d_{ij} \in [0, 1]$. Finally, assume that buyers and sellers know these probabilities and choose their search intensities accordingly. With

these additional assumptions, we are able keep the random search aspects of the model while accomodating the fact that we observe particular types of businesses doing business with one other with greater or lesser frequency than pure randomness would imply.

Success rates: For type- i buyers, the expected share of encounters that result in business partnerships is now:

$$a_i^B = \frac{\sum_j \sum_{n=0}^{n_{\max}} d_{ij} \sigma_j^S(n) P_j^S(n)}{\sum_j \sum_{n=0}^{n_{\max}} \sigma_j^S(n) P_j^S(n)} \quad (\text{A-11})$$

where $P_j^S(n) = H_j^S(n)/H^S$ is the share of matches that involve type- j sellers with n buyers.

Similarly, for type j sellers, the expected share of meetings that result in business partnerships is:

$$a_j^S = \frac{\sum_i \sum_{s=0}^{s_{\max}} d_{ij} \sigma_i^B(s) P_s^B(i)}{\sum_i \sum_{s=0}^{s_{\max}} \sigma_i^B(s) P_s^B(i)} \quad (\text{A-12})$$

where, recall, $P_i^B(s) = H_i^B(s)/H^B$ is the share of matches that involv type- i buyers who have s sellers. Thus, for a type- i buyer with s suppliers, the hazard of finding another compatible seller is $\sigma_i^B(s) a_i^B \theta^B$. Likewise, for a type- j seller with n buyers, the hazard of finding another compatible buyer is $\sigma_j^S(n) a_j^S \theta^S$.

Policy functions: Incorporating compatibility, the programming problem for a type- i buyer with s sellers becomes:

$$V_i^B(s) = \max_{\sigma_i^B(s)} \left\{ \frac{\pi_i^B(s) - c_B(\sigma^B) + s\delta V_i^B(s-1) + \sigma_i^B(s) a_i^B \theta^B V_i^B(s+1)}{\rho + s\delta + \sigma_i^B(s) a_i^B \theta^B} \right\} \quad (\text{A-13})$$

Accordingly, the new buyer policy functions, $\sigma_s^B(i)$, solve the first order conditions:

$$c'_B(\sigma_i^B(s)) = a_i^B \theta^B [V_i^B(s+1) - V_i^B(s)]. \quad (\text{A-14})$$

Similar modifications apply on the sellers' side. The value to a seller of an existing compatible relationship with a type- i buyer in state s now depends on a_i^B . This is because the hazard of this buyer adding another seller depends upon her compatibility:

$$V_{i,s}^S = \frac{\tau_i(s) + (s-1)\delta V_{i,s}^S(s-1) + \sigma_i^B(s)a_i^B\theta^B V_{i,s}^S(s+1)}{\rho + s\delta + \sigma_i^B(s)a_i^B\theta^B} \quad (\text{A-15})$$

And the ex-ante potential value of a new relationship with a compatible buyer is:

$$V_j^S = \sum_{i=1}^I \sum_{s=0}^{s_{\max}} V_i^S(s+1) \frac{d_{ij}P_i^B(s)}{\sum_{i,s} d_{ij}P_i^B(s)}$$

The associated seller policy functions, $\sigma_j^S(n)$, therefore solve:

$$c'_S(\sigma_j^S(n)) = a_j^S\theta^S V_j^S. \quad (\text{A-16})$$

Empirical implementation: The d_{ij} 's can be solved for using observed shares of different product categories at different firms, so this extension adds no new parameters to identify.

(Details to come.)